

# Designing Electronics for High Vibration and Shock

**Electronic equipment can be designed to withstand extreme mechanical stress, without the weight and cost penalties of overdesign.**

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The U.S. Air Force estimates that vibration and shock cause 20 percent of the mechanical failures in airborne electronics. Proper design procedures for ensuring equipment survival in a shock and vibration environment are therefore essential.

Interestingly, the remaining 80 percent of mechanical failures relate to thermal stresses induced by high thermal gradients, high thermal coefficients of expansion and a high modulus of elasticity.

In both cases, failures occur primarily from broken component lead wires, cracked solder joints, cracking of the component body, plated hole cracking, broken circuit traces, board delamination and electrical shorting.

Although this discussion will emphasize designing an electronic system for protection against damage from vibration and shock, designers must also be cognizant of those factors relating to thermally induced stresses.

Most modern electronic systems are composed of two major mechanical elements: an equipment chassis and a plug-in printed circuit board (PCB) assembly (Fig 1).

Extensive testing on various PCBs has determined the fatigue life of different components, component leads and solder joints, based on the dynamic displacement of the PCB. Mathematical equations exist to relate fatigue life with a PCB's natural frequency under sine-wave vibration, random vibration and shock. Therefore it is possible to design a PCB assembly to have a selected fatigue life coincident with its expected or predicted mechanical environment.

The dynamic properties of the chassis are also critical – especially the resonant frequency perpendicular to the plane of the PCB assembly – in that the chassis must not excessively amplify the vibration or shock input to the PCB. This is best accomplished by the “octave rule.”

For example, the natural frequency (resonant frequency) of the chassis must be a least one octave from the natural frequency of the PCB. Therefore if the PCB assembly has a frequency of 400 Hz, then the chassis must have a frequency no closer to 400 Hz than 200 Hz or 800 Hz.

### Vibration equations

Extensive PCB vibration testing has established that a fatigue life of about 10 million stress reversals under sinusoidal vibration can be achieved for lead wires and solder joints when the dynamic single amplitude displacement at the center of the board is limited to the value in Eq. 1. Similarly, about 20 million stress reversals can be achieved under random vibration.

$$Z = \frac{0.00022B}{ct\sqrt{L}} \quad (1)$$

where,

$Z$  = single amplitude dynamic displacement, in.;

$B$  = length of PCB edge parallel to component located at the center of the board (worse-case), in.;

$L$  = length of component, in.;

$t$  = thickness of PCB assembly, in.;

$c$  = 1.0 for standard DIP;

= 1.26 for a DIP with side-brazed leads;

= 1.26 for a pin-grid array with two parallel rows of pins;

= 1.0 for a pin-grid array with four rows of pins

(one row extending along the perimeter of each edge);

= 2.25 for leadless chip carriers.

The actual dynamic single amplitude displacement can also be given by:

$$Z = \frac{9.8 G_{out}}{f_n^2} = \frac{9.8 G_{in} Q}{f_n^2} \quad (2)$$

where,

$G_{in}$  = input acceleration to PCB, g's;

$f_n$  = resonant frequency or natural frequency of PCB, Hz;

$Q$  = transmissibility of PCB at its resonance.

Test data has shown that  $Q$  is related to the natural frequency as:

$$Q = \sqrt{f_n} \quad (3)$$

Equations 1, 2, and 3 can be combined to yield the required resonant frequency for achieving a fatigue life of 10 million stress reversals in a *sinusoidal vibration* environment.

$$f_n = \left[ \frac{9.8 G_{in} tc\sqrt{L}}{0.00022B} \right]^{2/3} \quad (4)$$

Likewise, the desired PCB resonant frequency can be determined that will yield a fatigue life of about 20 million stress reversals in a random vibration environment. The acceleration response is shown in Eq. 5 as the root mean square, or rms.

$$\sqrt{\frac{\pi}{2} P f_n Q} \quad (5)$$

where,

$G_{rms}$  = one-sigma rms acceleration response of PCB, g's;

$P$  = power spectral density input at  $f_n$   $g^2$ / Hz.

Combining Eqs. 1, 2, 3 and 5 yields the desired PCB resonant frequency for achieving about 20 million stress reversals as a fatigue life in a *random vibration* environment. Equation 6 takes into account 3-sigma acceleration peaks.

$$f_n = \left[ \frac{29.4tc \sqrt{\frac{\pi}{2} P L}}{0.00022B} \right]^{0.8} \quad (6)$$

### Mechanical shock

It has been determined that in a shock environment of less than a few thousand total cycles, the allowable stress levels can be up to six times higher than allowable vibration stress levels. Therefore, multiplying Eq. 1 by Eq. 6 yields:

$$Z = \frac{0.00132B}{ct\sqrt{L}} \quad (7)$$

Also, the actual shock displacement expected at the center of the PCB can be expressed as:

$$Z = \frac{9.8 G_{in} A}{f_n^2} \quad (8)$$

where  $A$  = shock amplification factor, typical values are between 0.5 and 1.5.

Combining Eqs. 7 and 8 yields the required PCB natural frequency that minimizes the probability of structural failure under mechanical shock when a limited number of shocks are expected.

$$f_n = \left[ \frac{9.8 G_{in} A t c \sqrt{L}}{0.00132B} \right]^{0.5} \quad (9)$$

### Estimating fatigue life

The fatigue life of the various materials used for components, leads and PCBs is related to the resonant frequency of the assembly, time under vibration test, stress level and fatigue properties of the materials. A log-log plot of stress levels versus fatigue cycles yields Eq. 10 as the relation between the stress and the number of cycles to fail.

$$N_1 S_1^b = N_2 S_2^b \quad (10)$$

where,

$N_1$  = number of stress cycles to produce a failure at stress level number 1;

$N_2$  = number of stress cycles to produce a failure at stress level number 2;

$S_1$  = magnitude of stress for failure at level 1;

$S_2$  = magnitude of stress for failure at level 2;

$b$  = fatigue exponent represented by the slope of the plotted curve: typical value is 6.4.

For linear systems, the stress level is directly related to the displacement ( $Z$ ) and also to the acceleration ( $G$ ). The number of stress reversals is related to the test time ( $T$ ). Therefore, Eq. 10 can be modified as:

$$T_1 Z_1^b = T_2 Z_2^b \quad (11)$$

$$T_1 G_1^b = T_2 G_2^b \quad (12)$$

A general rule in approximating the required fatigue life of a system is to set the design goal for *a fatigue life equal to five vibration/shock qualification tests*. This is especially true for airborne systems where weight savings are important and where overdesign is to be avoided.

Designing a system to survive a high vibration and shock environment is illustrated in the following example.

### Spaceborne computer

The computer shown in Fig. 1 is for a spacecraft application and was designed and built by the Guidance and Control Systems Division of Litton Industries. A typical printed circuit board assembly is shown in Fig. 2. The assembly consists of multilayer boards bonded to each side of an aluminum heat-sink plate. Dimensions are shown in Fig. 3.

The computer has been designed for reliable operation while exposed to a white-noise random vibration environment of  $0.12 \text{ g}^2/\text{Hz}$ , over a frequency bandwidth of 20 to 2,000 Hz, with an acceleration level of  $15.4 \text{ g}'\text{s rms}$  for a period of 15 minutes in each of three axis, for a total test time of 45 minutes. The system must also withstand 18 shocks of  $450 \text{ g}'\text{s}$  with a half-sine shock pulse of 0.0005 s, with three shocks in two directions along each of the three axis.

The various steps in determining the adequacy of the design consist of first calculating the required resonant frequency of the PCB assembly as to component lead fatigue life under vibration and shock, then comparing the required resonant frequency with that of the actual resonant frequency expected when the assembly is mounted in the chassis.

*Step 1.* Determine the desired resonant frequency of the PCB assembly for the random-vibration environment.

The largest components on the circuit boards – hence, exhibiting the most lead stress under board flexure – are the side-brazed 40-pin DIPs. The desired resonant frequency for a fatigue life of 20 million stress cycles can be found from Eq. 6:

$$f_n = \left[ \frac{29.4(0.14)(1.26) \sqrt{\frac{\pi}{2} (0.12)(2)}}{0.00022(5.5)} \right]^{0.8}$$

$$= 544 \text{ Hz}$$

where,

$$t = 0.14;$$

$$c = 1.26;$$

$$P = 0.12;$$

$$L = 2.0; \text{ and}$$

$$B = 5.5.$$

Note that the 40-pin DIP was not mounted parallel to the short side (4.24 in.) of the PCB, but rather the vertical long side (5.5 in.), because the former would expose the component to greater flexure over its length.

*Step 2.* Determine the expected PCB vibration fatigue life and compare it to the desired vibration fatigue life.

The approximate fatigue life expected for the PCB assembly can be determined from:

$$\text{Life} = \frac{20 \times 10^6 \text{ cycles to fail}}{544 \text{ cycles / s (3600 s / h)}} \quad \text{Life} = \frac{20 \times 10^6 \text{ cycles.to.fail}}{544 \text{ cycles / sec (3600s / h)}} = 10.2 \text{ h}$$

$$= 10.2 \text{ h}$$

However, when the desired random-vibration fatigue life requirement is limited to five qualification tests, then the desired fatigue life will be:

$$5 \text{ tests} \times 45 \text{ min./test} = 225 \text{ min.}$$

$$= 3.75 \text{ h}$$

Therefore it is not necessary for the PCB assembly to have a resonant frequency of 544 Hz to meet the vibration requirement. A lower frequency will be acceptable if it provides a fatigue life of 3.75 h.

*Step 3.* Determine the resonant frequency required to obtain a fatigue life of 3.75 h under random vibration.

Equation 11 can be used to find the required resonant frequency.

$$T_1 Z_1^b = T_2 Z_2^b$$

Before this equation can be used, the dynamic displacement of the PCB assembly must be determined, first by using Eq. 5 to obtain the rms acceleration g level, then using Eq. 2 to obtain the dynamic displacement. First, Eq. 5:

$$G_1 = \sqrt{(1.57)(0.12)(544)(23.3)}$$

$$= 48.8 \text{ rms}$$

where,

$$P = 0.12 \text{ g}^2 / \text{Hz}$$

$$f_n = 544 \text{ Hz}$$

$$Q = \sqrt{544} = 23.3$$

Substituting into Eq. 2 for finding  $Z_1$ :

$$Z_1 = \frac{9.8(48.8)}{(544)^2}$$

$$= 0.001618 \text{ in. rms}$$

Using Eq. 5 again for the required resonant frequency:

$$G_2 = \sqrt{(1.57)(0.12)(f_2)(f_2)^{(0.5)}}$$

$$= 0.434 (f_2)^{0.75}$$

Substituting again in Eq. 2 for  $Z_2$ :

$$Z_2 = \frac{9.8(0.434)(f_2)^{0.75}}{(f_2)^2}$$

$$= 4.253 (f_2)^{-1.25}$$

Also,

$$T_1 = 10.2 \text{ h of fatigue life}$$

$$T_2 = 3.75 \text{ h of fatigue life}$$

Substituting all into Eq. 11 yields:

$$10.2(0.001618)^{6.4} = 3.75 [4.253(f_2)^{-1.25}]^{6.4}$$

$$f_2 = 480 \text{ Hz}$$

Therefore, the components on the PCB assembly will have a fatigue life of 3.75 h when the assembly has a resonant frequency of at least 480 Hz.

*Step 4.* Determine the desired resonant frequency for the shock environment.

Eq. 9 can be used to find the resonant frequency. However, the shock amplification factor ( $A$ ) must first be found by:

$$A = 2R$$

where,

$$R = \frac{f_n \text{ of PCB assembly}}{f_p \text{ shock frequency}}$$

The shock pulse frequency for the half-sine shock can be obtained by doubling the pulse duration to find the period for the full sine wave in s/cycle. This is then inverted to obtain the shock pulse frequency in cycles/s.

$$\begin{aligned} f_p &= \frac{1}{2 (0.0005)} \\ &= 1000 \text{ Hz} \end{aligned}$$

The resonant frequency of the PCB assembly is not known and must be assumed in order to find ( $R$ ) and then calculated from Eq. 9. If the comparison between the assumed value and the calculated value is poor, another value can be assumed until a good comparison is obtained. For example, assuming 400 Hz for  $f_n$  in finding ( $R$ ) would yield a result of 348 Hz from Eq. 9, a poor match.

Assuming  $f_n = 300$  Hz, then:

$$R = \frac{300}{1000} = 0.3 \quad \text{and:}$$

$$A = 2(0.3) = 0.6$$

then, from Eq. 9:

$$\begin{aligned} f_n &= \left[ \frac{9.8(450)(90.6)(0.14)(1.26)\sqrt{2}}{(0.00132)(5.5)} \right]^{0.5} \\ &= 301 \text{ Hz (a good match)} \end{aligned}$$

The required resonant frequency of 301 Hz for the shock environment must now be compared with the required resonant frequency for the random vibration environment to see which environment dictates the design. Since the resonant frequency of 480 Hz for random vibration is higher than 301 Hz, the 480 Hz dominates and will satisfy both the shock and vibration requirements (the higher the resonant frequency, the lower the PCB displacement and the lower the stress).

*Step 5.* Determine the resonant frequency of the PCB assembly when mounted in the chassis.

The plug-in PCB assembly within the chassis consists of two wedge-clamps for holding the aluminum plate in the chassis slot. The top edge is free and the bottom is restrained by a connector. Experience has shown that flat-plate deflection theory is applicable for this situation in the form of:

$$f_n = \frac{\pi}{1.74} \left[ \frac{D}{M} \left( \frac{4}{a^4} + \frac{1}{2a^2b^2} + \frac{1}{64b^4} \right) \right]^{0.5}$$

where,

$D$  = plate stiffness factor,  $EI/\text{in.}$ ;

$M$  = mass/area;

$a$  = PCB width, in.;

$b$  = PCB height, in.                      also:

$$EI = \frac{E_e(h_a^3 - h_i^3)}{12(1 - u_e^2)} + \frac{E_a h_i^3}{12(1 - u_a^2)}$$

where,

$E_e = 2 \times 10^6$  psi epoxy modulus of elasticity;

$E_a = 10 \times 10^6$  psi aluminum modulus of elasticity;

$h_a = 0.14$  in., assembly thickness;

$h_i = 0.04$  in., aluminum thickness;

$u_e = 0.12$ , Poisson's ratio for epoxy;

$u_a = 0.30$ , Poisson's ratio for aluminum.

Solving for  $EI$  yields:

$EI = 512$  lb/in. for a 1-in. section, then,

$$D = \frac{512}{\text{inch}} = 512 \text{ lb/in.}$$

$$M = \frac{w}{g a b} = 0.0000944 \text{ lb-s}^2/\text{in.}^3$$

where,



$w = 0.85$  lb, PCB assembly weight;  
 $g = 386$  in./s<sup>2</sup>, acceleration of gravity.

Substituting and solving for  $f_n$  yields:

$$f_n = 485 \text{ Hz}$$

The resonant frequency of the mounted PCB assembly is therefore 485 Hz and is slightly greater than the required frequency of 480 Hz, hence is satisfactory for the random-vibration environment.

### **Chassis frequency**

The natural frequency of the chassis must be at least one octave from the PCB assembly's frequency to minimize dynamic coupling. In this case, the chassis should be  $480/2 = 240$  Hz or  $480 \times 2 = 960$  Hz. Which value to choose is often arbitrary, however, the choice should involve a consideration of the best frequency for minimizing the shock amplification factor. This can be done by plotting the frequency ratio of the PCB assembly and chassis ( $f_n \text{ PCB}/f_n \text{ chassis}$ ) versus the amplification factor ( $A$ ) and then noting at which of the two frequencies the curve value for ( $A$ ) is at a minimum.

Finding the actual resonant frequency of the chassis can be handled by building a prototype for testing or by complex computer modeling. For calculation purposes, the following equation can be used to find a rough approximation for the chassis'  $f_n$  perpendicular to the plane of the PCBs.

$$f_n = \frac{1}{2\pi} \sqrt{\frac{g}{Y}}$$

where,

$g$  = acceleration of gravity; and

$Y$  = static displacement due to chassis weight.

REF: Vibration Analysis For Electronic Equip. 1st. ed. 1973, 2nd. ed. 1988, 3rd. ed. 2000 by Dave S. Steinberg, published by John Wiley & Sons Inc.

Dave was the Manager of the Mechanical Engineering Department at Litton GCS. Dave is presently a consultant, author of textbooks on vibration, cooling and thermal cycling fatigue. He is president of Steinberg & Associates and presents worldwide seminars on vibration, shock and thermal management. He is also a Professor at UCLA where he teaches full semester post graduate courses on the above subjects using his textbooks.

NOTE: This article may be a little difficult to follow because the figures referenced in the article were too dark to reproduce, so they were left out. We hope the information presented here will still be of use to people interested these electronic packaging subjects.